Enforcing deterministic polarisation in a reverberating environment

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A report is presented on a technique for generating coherently-polarised pulsed fields within highly reverberating environments. The ability of doing so is predicted theoretically, showing that the purity of the polarisation of the electromagnetic field does not depend on the cross-polarisation rejection of the source antenna, but only on the well-known depolarisation properties of standard reverberation chambers. Experimental results are provided, proving that the theoretical model is sound, thus validating the first technique for generating a coherent arbitrarily polarised field in a reverberating environment.

Introduction: The success of standard reverberation chambers (RCs) as an electromagnetic compatibility facility is mainly due to two features: (i) the equipment under test is submitted by a large number of plane waves whose random directions of propagation and polarisations can be changed almost instantly through modal stirring, thus allowing the likely excitation of all of its weaknesses, and (ii) high-intensity fields can be generated from low-power sources. Nevertheless, the field they generate cannot be set in a deterministic way, and only its statistical moments are known [1]. In many applications it would be useful to be able to enforce a deterministic polarisation, while keeping point (ii). This is not feasible in standard RCs, owing to the strongly incoherent nature of the field polarisation. This notwithstanding, time-reversal techniques have been proven to be capable of enforcing deterministic properties in intrinsically complex and random media, as long as losses are low and the system is time invariant [2]. An example of this ability is given in Fig. 1, where a pulsed field is transmitted through a reverberation chamber, and compared to the desired waveform. In this Letter, we prove for the first time that polarisation coherence can also be reinstated, showing that it can be controlled with no limitations by simply modifying the excitation signal applied to the transmitting antenna, thus allowing a real-time coherent control of the field, with no need of either mechanical movements, or of antenna arrays.

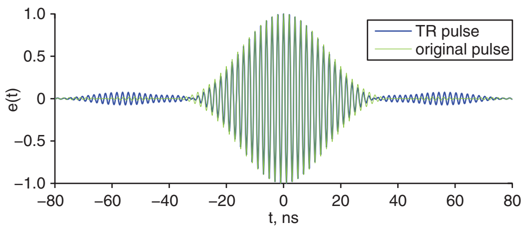

Fig. 1 *Example of ability of time-reversal techniques to generate coherent pulses in reverberating environment*

Light grey curve represents pulse received at given location in RC when applying time-reversal techniques, computed by means of experimentally measured transfer function. Black curve is original pulse to be transmitted. Two peak-normalised curves are indistinguishable around peak region

Asymptotic polarisation properties: We consider the same setup as for standard RC applications, i.e. a transmitting antenna placed within the RC in order to excite a field distribution. The vector electric field $\mathbf{E}(f, \mathbf{r})$ measured at any point $\mathbf{r}$ inside the RC can then be related to the signal $X(f)$ applied to the antenna as

$$\mathbf{E}(f,\mathbf{r}) = X(f)\boldsymbol{\Phi}(f,\mathbf{r}) = X(f)\begin{pmatrix}\Phi_x & \Phi_y & \Phi_z\end{pmatrix}^{\mathrm{T}}(f,\mathbf{r}) \qquad (1)$$

where $\boldsymbol{\Phi}(f, \mathbf{r})$ is a vector transfer function, made up of three scalar transfer functions related to each Cartesian polarisation component; these will be referred to as $\Phi_i(f, \mathbf{r})$ with $i = 1, \ldots, 3$ for, respectively, the x, y and z components. It is known that for an overmoded RC, the $\Phi_i(f, \mathbf{r})$ transfer functions are submitted to the following orthogonality condition [3]:

$$\mathrm{E}[\Phi_i(f,\mathbf{r})\Phi_j^*(f,\mathbf{r})] = C\delta_{ij} \qquad (2)$$

where $\mathrm{E}[\cdot]$ is the expected value operator and C is a normalisation constant. This condition is satisfied only when averaging over the entire space of the random realisations of the transfer functions, e.g. such as when applying mode-stirring techniques. By recalling the modal theory underpinning the resonant phenomena occurring in an RC, a generic scalar transfer function can be expressed as

$$\Phi(f) = \sum_{i=1}^{M} \gamma_i \psi_i(f) \qquad (3)$$

where $\psi_i(f)$ is the frequency response of the ith resonant mode supported by the RC, centred around the frequency f_i, while $\gamma_i \in \mathbb{C}$ models how it is excited. Equation (3) is defined over a bandwidth B_T centred around f_0, where the RC supports M modes. Let us now assume that the $\{f_i\}$ and $\{\gamma_i\}$ are ergodic random processes, so that the average ensemble operator can be approximated through the arithmetic mean as applied to the different modes defining any transfer function. Recalling (2), the law of large numbers would then imply that

$$\lim_{M\to\infty}\int_{B_T}\Phi_i(f)\Phi_j^*(f)\mathrm{d}f = M\mathrm{E}[\Phi_i(f_0)\Phi_j^*(f_0)] \qquad (4)$$

considering the equality as a convergence in probability. Equation (4) is the cornerstone of the proposed method, since it implies that the same performance that would be obtained only by averaging over a large number of random realisations, can be fairly approximated when using wideband signals in a single deterministic configuration, provided that the RC be in an overmoded state. This feature is in particular related to the self-averaging properties of time-reversal, as investigated in [2].

Having introduced ergodicity and (4), we can now describe how a coherent deterministic polarisation can be enforced. Let us consider an excitation signal $X_{\mathrm{TR}}(f)$ defined as

$$X_{\mathrm{TR}}(f) = G(f)\sum_{i=1}^{3}p_i\Phi_i^*(f) = G(f)\boldsymbol{\Phi}^{\mathrm{H}}\mathbf{p} \qquad (5)$$

where $G(f)$ is the spectrum of the pulse $g(t)$ to be generated at $\mathbf{r}$, with bandwidth B_T, and H is the Hermitian operator, while $\mathbf{p} = (p_1\, p_2\, p_3)^{\mathrm{T}}$ is a vector containing the complex weights of the desired polarisation pattern to be enforced. Applying the signal (5) to (1) yields a received field

$$\mathbf{E}_{\mathrm{TR}} = G\boldsymbol{\Phi}\boldsymbol{\Phi}^{H}\mathbf{p} \qquad (6)$$

having dropped the function arguments for the sake of simplicity. Since we are rather interested in the time-domain field, and especially over the peak of the pulse at $t = 0$, we get

$$\mathbf{e}_{\mathrm{TR}}(0) = \int_{-\infty}^{+\infty} = G\boldsymbol{\Phi}\boldsymbol{\Phi}^{\mathrm{H}}\mathbf{p}\mathrm{d}f = \sqrt{\boldsymbol{\mathcal{E}}}\boldsymbol{\rho}\sqrt{\boldsymbol{\mathcal{E}}}\mathbf{p} \qquad (7)$$

having introduced the energy matrix $\boldsymbol{\mathcal{E}} = \mathrm{diag}\{\mathcal{E}_1, \ldots, \mathcal{E}_3\}$, with

$$\mathcal{E}_i = \int_{-\infty}^{+\infty} G|\Phi_i|^2\mathrm{d}f = 2\int_{B_T}\mathrm{Re}\{G\}|\Phi_i|^2\mathrm{d}f \qquad (8)$$

and the polarisation matrix $\boldsymbol{\rho}$, whose elements are defined as

$$\rho_{ij} = \frac{2\int_{B_T}\mathrm{Re}\{G\Phi_i\Phi_j^*\}\mathrm{d}f}{\sqrt{\mathcal{E}_i\mathcal{E}_j}} \qquad (9)$$

By applying (4), it can be proven that

$$\lim_{M\to\infty}\boldsymbol{\rho} = \mathrm{E}[\boldsymbol{\rho}] = \mathbf{1} \qquad (10)$$

where $\mathbf{1}$ is the identity matrix. Recalling that in an overmoded RC the field is statistically isotropic, i.e. $\mathrm{E}[|\Phi_i|^2] = \mathrm{E}[|\Phi_j|^2], \forall i, j$, by applying (4) to this last equation too, $\lim_{M\to\infty}\mathcal{E}_i = \mathcal{E}_0, \forall i$. We can hence claim that

$$\lim_{M\to\infty}\mathbf{e}_{\mathrm{TR}}(0) = \mathcal{E}_0\mathbf{p} \qquad (11)$$

This result proves that without invoking any statistical averaging process, i.e. no stirring, the pulsed field generated through time-reversal converges, for a sufficiently overmoded RC, to a deterministic coherently polarised field, directly controlled by the weight vector $\mathbf{p}$, and this for any static configuration. In other words, the Φ_i functions approximate an orthogonal basis. This result has been derived as an asymptotic property, so that the actual received field is expected to fulfil (11) on average, while presenting a statistical dispersion inversely dependent on M.

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Experimental results: Experimental validation tests were carried out in Supélec's RC ($3.08 \times 1.84 \times 2.44$ m^3), using a log-periodic dipole antenna (LPDA) positioned near one corner of the chamber, with the dipoles of the antenna aligned along the vertical direction (z-axis), while the direction of maximum gain was aimed at the corner. Concerning the receiving transducer, an all-optical E-field probe was used, manufactured by Enprobe, model EFS-105. This phase-preserving probe is linearly polarised, with a cross-polarisation rejection of about 40 dB, allowing accurate measurement of the cross-polarisation of the received pulse. The probe was mounted over a styrofoam support, designed to ensure measurement of the three Cartesian components of the E field. A total of 50 positions were considered, scattered uniformly over the lower half of the RC; for each of these, the transfer functions between the LPDA and the probe was measured along the three polarisations, by means of a vector network analyser. Three frequencies were considered for f_0, namely 1, 1.5 and 2 GHz, considering a bandwidth $B_T = 100$ MHz. The pulse $g(t)$ was set to be a Gaussian pulse, with a -20 dB frequency bandwidth B_T.

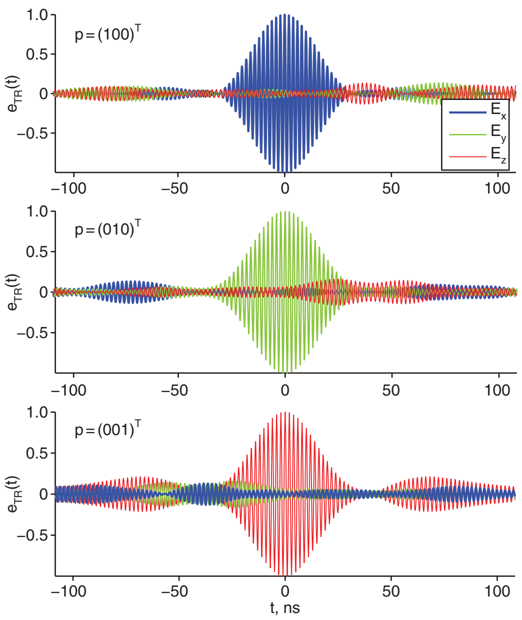

Fig. 2 *Field components obtained from experimental results measured at one position, for Gaussian pulse at 1.5 GHz*

Each plot refers to weight vector p corresponding to one Cartesian direction. Top to bottom, x, y, z components of fields are ideally only excited when pulse attains peak value

From the spectrum of $g(t)$ and the transfer functions, the energy matrix and the polarisation matrix $\boldsymbol{\rho}$ were computed, as defined in (8) and (9), respectively. We first checked the validity of the isotropy assumption, by computing how the energy received along the three polarisations is distributed. The first two statistical moments were computed, and are shown in Table 1, proving that this assumption makes sense for the three frequencies we chose, with a maximum error on the average energy of about 8% and an average one of 5%. A similar statistical analysis was carried out on the off-diagonal elements of $\boldsymbol{\rho}$: the results shown in Table 1 prove that indeed the field components orthogonal to the originally addressed one are on average very close to zero. These results prove that the ergodic assumption is indeed valid. Time-domain results are shown in Fig. 2, showing how the three Cartesian linear polarisations can be separately addressed by means of the proposed method.

Concerning the standard deviation of the rejection, it is directly related to the residual error when considering a finite number of modes. Nevertheless, it does not change much when doubling f_0. This is due to the limited number of degrees of freedom actually available when the quality factor Q of the modes is finite: it was indeed demonstrated in [4] that of M modes available, a maximum of about $M_e = B_T Q / f_0$ are actually independent. This interpretation is supported by the inverse trends followed by the standard deviation and M_e, as shown in Table 1.

Table 1: Statistics of performance in pulse transmission obtained from collected experimental data (average values presented for energy matrix and off-diagonal terms of matrix $\boldsymbol{\rho}$, with standard deviations given in parentheses

f	1.0 GHz	1.5 GHz	2.0 GHz
$\mathcal{E}_1$	0.94 (0.12)	0.96 (0.13)	0.96 (0.11)
$\mathcal{E}_2$	0.92 (0.13)	0.94 (0.12)	0.97 (0.12)
$\mathcal{E}_3$	1.00 (0.14)	1.00 (0.14)	1.00 (0.14)
ρ_{12}	0.025 (0.056)	0.003 (0.061)	0.013 (0.074)
ρ_{13}	0.014 (0.045)	0.004 (0.061)	0.018 (0.076)
ρ_{23}	−0.021 (0.068)	−0.024 (0.070)	−0.012 (0.081)
M_e	570	420	315

Conclusions: We have introduced the first method for enforcing a coherent and deterministic polarisation upon pulsed fields transmitted in a highly-reverberating environment. This novel approach is based jointly on the properties of time-reversal techniques and the strong depolarisation experienced in reverberating media. In particular, we have proven that the polarisation of the field can be controlled in a precise way by simply operating on the signal applied to the excitation antenna. Experimental results support this analysis, demonstrating that actual applications can be defined, such as high-power microwave testing with real-time polarisation modification.

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